Test paper 1 Dynamics of Ocean Structures

Maximum marks: 40 Time: 3 Hrs

Please read the following instructions carefully before answering the paper

This question paper has 3 sections namely A, B and C. All sections should be attempted to qualify your answer script for evaluation. Figures for relevant questions are given at the end

Each section has one additional bonus question. Correct answer to this question shall fetch you extra, bonus marks.

Any data found missing may be suitably assumed and stated in the problem.

All numerical should be solved strictly in accordance to SI units only.

Draw neat figures/ sketches wherever necessary to support your answer

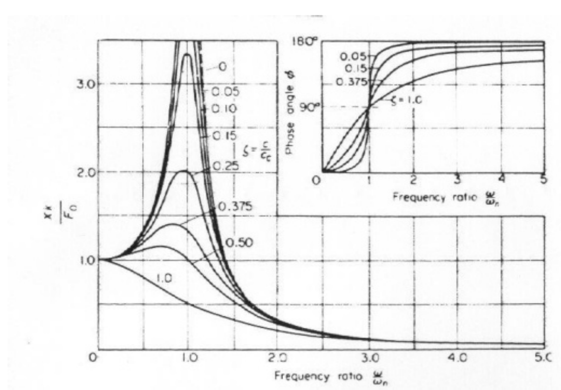
Answer the questions to the point. Be brief. Round about answers shall fetch you a negative score.

SECTION A (Answer any ten questions; each question carries one mark)

1. An alternate approach which states that *the system may be set in dynamic equilibrium* is called as ________
2. An experimental observation showed that the amplitude of free vibration of a beam, modeled as a single degree of freedom decreases from 1 to 0.2 in 5cycles. What is the % of critical damping?
3. Define *Octave*.
4. What is beating phenomenon? Discuss its severity in structures subjected to dynamic loading
5. Compare the response build-up of a spring-mass system at resonant excitation, when the system is in the damped and undamped state. Draw neat sketches to explain the answer.
6. Total response of an idealized single degree-of-freedom has two components namely: _________ and _________
7. Write a brief note on viscous damping and its application in offshore structures.
8. Name at least three numerical methods used to estimate natural frequencies and mode shapes of multi- degree-of-freedom system.
9. Explain why an iterative procedure should converge to first mode analysis?
10. A circular pulley of mass, m and radius, r is connected by a spring of modulus, k. (ref Fig. 1). The pulley is free to roll on the rough horizontal surface without any slip. Find its ω_n. Use energy method.
11. Mass, m is hanging from a cord attached to a circular disc of mass, M and radius, R. Disc is restrained from rotation by a spring attached at 'a' from its center. If the suspended mass, m is displaced downwards from its rest position, find ω_n (Ref Fig. 2).
12. In fluid-structure interaction, total damping forces arise from four sources namely: __________, _________, __________ and __________.

Bonus question

List the inferences you draw from the adjacent figure



SECTION B (Answer any five questions; each question carries 2 marks)

1. Show that the log decrement is also given by the equation $\delta = \frac{1}{n} \ell n \left(\frac{x_0}{x_n} \right)$ where x_n represents amplitude after n cycles have elapsed. Plot also the curve showing the # of cycles elapsed against ξ for the amplitude to diminish by 50%

2. For f(t) be arbitrary periodic loading as applied to single degree-of-freedom system, direct solution by possible differential equation approach is not possible; for such a system, assume a prescribed dynamic loading and derive the solution. (*Hint: chose an arbitrary impulse function as input load*).

3. Derive the frequency response function H(ω) for the system whose behavior is determined in terms of system impulse response h(t) used in the convolution integral.

4. List the guidelines for modal truncations in multi degree-of-freedom system models. In modal combination rules, how participation from the cross modes are accounted for? In such cases, how do you estimate the net response by accounting for participation from higher modes? Explain briefly.

5. Define *wake region* in the context of fluid-structure interaction.

6. A cantilever beam of length, ℓ and rigidity EI supports a weight of $3W_0$ at its free end. At t=0, one-third of the weight is suddenly removed. What will be the equation for the displacement at the free end at any time thereafter?

7. Derive the equivalent viscous damping for coulomb damping

8. A vibrating system with mass 5 kg and stiffness 3000 N/m is viscously damped so that the ratio of two consecutive peaks is reduced from 1 to 0.85. Determine the natural frequency, log decrement, damping ration, damping coefficient and damped natural frequency.

9. Determine two frequencies of the spring –mass system shown in Fig. 3. Use Rayleigh-Ritz method only.

Bonus question

Discuss the variation of modal damping ratio with natural frequency. (*Hint: consider mass-proportional damping, stiffness proportional damping and Rayleigh damping*)

SECTION C (Answer any four questions; each question carries five marks)

1. Derive the influence coefficient matrix and determine all the frequencies and mode shapes of the spring-mass system shown in Fig. 4.

2. Derive equations of motion and influence coefficient matrix of the branched system shown in Fig. 5

3. Determine the fundamental frequency and mode shape of the system shown in Fig. 6. Use Holzer method only. Take $m_1=m_2=m_3=m=2000$ kg; $k_1=k_2=k_3=k=1500$ kN/m.

4. An offshore triangular Tension Leg Platform is analyzed for seismic forces at the sea-bed, to which its tethers are anchored. TLP is idealized as a rigid body having six degrees-of-freedom namely three translational (along X,Y and Z) and three rotational degrees-of-freedom (about X, Y and Z axes). Stiffness matrix of TLP is given below. It can be seen that the stiffness coefficients are function of platform displacements and variation in tether tension as well. Write the elements of mass matrix, including the components of added mass arising from the variable submergence effect. Write down the equation of motion of the platform to analyze it under wave loads and earthquake forces. Clearly explain the method by which dynamic analysis of TLP under earthquake forces can be carried out. Take damping matrix as proportional to mass and stiffness matrix. Use ξ = 2%.

List the nonlinearities encountered in this analysis. How do you propose to solve the equations of motion of such systems? Explain why de-coupling of equations of motion is not possible in this case?

$$K = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 \\ k_{31} & k_{32} & k_{33} & k_{34} & 0 & k_{36} \\ 0 & k_{42} & 0 & k_{44} & 0 & 0 \\ k_{51} & 0 & 0 & 0 & k_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{66} \end{bmatrix}$$

$$k_{11} = \frac{3(T_0 + \Delta T_1)}{\sqrt{x_1^2 + \ell^2}}$$

$$k_{22} = \frac{3(T_0 + \Delta T_2)}{\sqrt{x_2^2 + \ell^2}}$$

$$k_{31} = \frac{3T_0(\cos\gamma_x - 1) + 3\Delta T_1 \cos\gamma_x}{x_1}$$

$$k_{32} = \frac{3T_0(\cos\gamma_y - 1) + 3\Delta T_2 \cos\gamma_y}{x_1}$$

$$k_{51} = -k_{11}\bar{h}$$

$$k_{42} = -k_{22}\bar{h}$$

$$\Delta T_1 = \left[\sqrt{x_1^2 + \ell^2} - \ell\right]\frac{AE}{\ell}$$

$$\Delta T_2 = \left[\sqrt{x_2^2 + \ell^2} - \ell\right]\frac{AE}{\ell}$$

$$k_{33} = 3\frac{AE}{\ell} + \frac{3}{4}\pi\rho g D_c^2$$

$$k_{34} = \frac{AE}{\ell}\frac{P_\ell}{2}\cos\theta_4$$

$$k_{44}\theta_4 = (F_b + T_0 + \Delta T_4)\bar{h}\sin\theta_4$$

$$k_{55}\theta_5 = F_b\bar{h}\sin\theta_5$$

$$k_{36} = 3T_0\left[\frac{\ell}{\ell_1} - 1\right] + 3\Delta T_6\left[\frac{\ell}{\ell_1}\right]$$

$$k_{66} = 3\frac{(T_0 + \Delta T_6)2a^2}{\ell_1}$$

$$a = \left[\frac{P_\ell}{2}\right]^2 + \left[\frac{P_b}{3}\right]^2$$

$$\ell_1 = \sqrt{\ell^2 + \theta_6^2(2a^2)}$$

$$\Delta T_6 = \frac{AE}{\ell}(\ell_1 - \ell)$$

where A is the cross-section area of the tether, E is the Young's modulus of the tether, ΔT_i is the increase in the initial pretension due to arbitrary displacement given in i[th] degree-of-freedom, ℓ is the length of the tether, x_i is the arbitrary displacement in i[th] degree-of-freedom, $\bar{h}$ is the distance of CG from the base of the pontoon, P_b, P_ℓ are plan dimensions of TLP normal and parallel to wave direction, respectively.

5. Explain half-power band width method which is used to estimate damping in a structural system. Derive the equation to estimate ξ for the known frequencies (f_1, f_2) in a particular band width.

Bonus question

For [K] and [M] be symmetric matrices of stiffness and mass of a given structural system, prove the orthogonality principle with respect to [K] and [M] respectively. Also, derive generalized mass and stiffness matrix for i[th] mode.

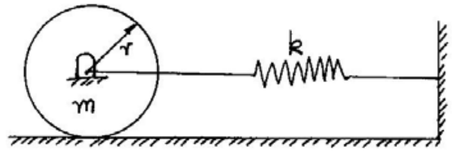


Fig. 1 (question A-10)

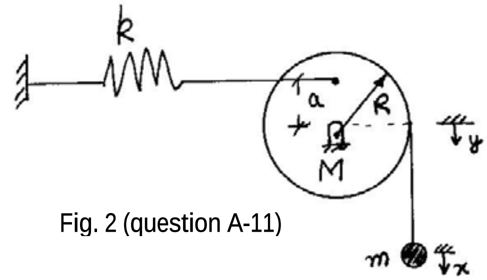


Fig. 2 (question A-11)

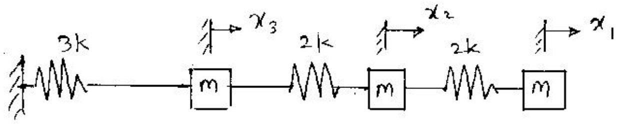


Fig. 3 (question B-9)

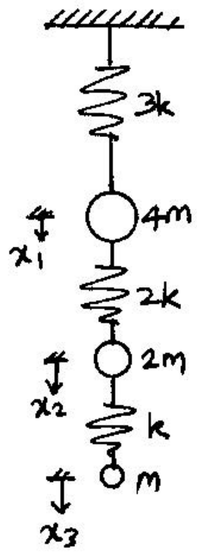


Fig. 4 (question C-1)

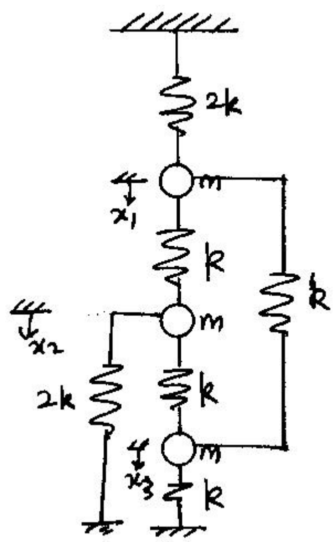


Fig. 5 (question C-2)

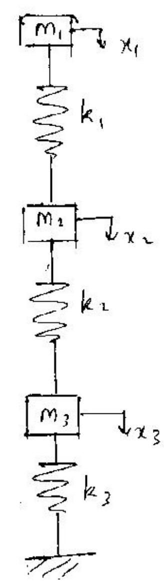


Fig. 6 (question C-3)